

Generalization of Goldbach's Conjecture

We can generalize / extend the Goldbach's weak (ternary) conjecture (that is proven by Helfgott) in two ways.

1. We can replace the primes as elementary building blocks with other types of numbers. e.g. with lucky numbers. The rules we follow to build natural numbers are the same (we use exclusively only additions). It is the more common approach.
2. The other possibility is to change those rules that prescribe the handling of elementary building blocks (numbers). The Goldbach conjecture is based on repeated additions, but we can try other combinations of elementary operations as well. The conclusion is that the Goldbach conjecture seems to be true for any possible combination of elementary operations where logically is not impossible to generate all the positive integers.

Using three primes and basic mathematical operations (+, -, *, :) as building blocks, these are the possible combinations to try to create all integers:

combination:	generates all odd numbers?	
$p+p+p$	yes	(Goldbach)
$(p+p)-p$	yes	
$(p-p)-p$	yes	
$(p-p)+p$	yes	
$(p*p)+p$	yes	
$(p*p)-p$	yes	
$(p+p)*p$	not	
$(p-p)*p$	not	
$(p:p)*p$	not	
$p*p*p$	not	
$(p:p)+p$	not	
$(p:p)-p$	not	
$(p:p):p$	not	

The conjecture is that

- if (from left to right) the first mathematical operation is division, OR
- if the second mathematical operation is multiplication then: it is impossible to build every number using only primes

(obviously, the result of the division of a prime with another prime is not an integer; and the multiplication with a prime number results that certain numbers (odd numbers) are unreachable).

BUT: if these exclusionary reasons does not exist or, in other words:

if the logic of the mathematical operations does not exclude, it is possible to construct every positive integer (larger than a certain small number) using maximum three primes or another building blocks.

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Links:

Helfgott, H. A. "The Ternary Goldbach Conjecture Is True." Jan. 17, 2014.

<https://arxiv.org/pdf/1312.7748.pdf>

Lucky number: <http://demonstrations.wolfram.com/LuckyNumbers/>

Sum of Divisors Conjecture:

https://www.researchgate.net/publication/317184248_Sum_of_Divisors_Conjecture

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