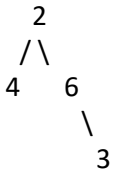
Graph numbers of divisors

According to our conjecture, every number can be classified into four groups where the classification is based on the features of the divisibility graphs.

Graph as the representation of divisibility

The numbers can be represented as graphs where the divisors are the nodes and the divisibilities are the edge. Both the number itself and 1 are ignored. Instead of d(n), d(n-2) represents the numbers of the divisors.
For example, if the number is 12, then



4 and 6 are divisible by 2; 6 is divisible by 2 and 3. Regarding a neighbour of a node, if the node is interconnected to other node(s), then one node is divisible by other(s). The edges aren't ordered.
The figure above can be displayed as 4-2-6-3.
So we can use this notation to display the relations between the divisors of 12:
2-4-6 (connect 2 with 4 and 6)
3-6 (connect 3 with 6)
If you follow these instructions, they result the whole graph.

Classification

We can classify all possible graphs of the numbers of divisors into four categories:

CG or complete graph:
powers of primes (p^k, p prime, k >= 1). They contain every possible edges. Obviously, this category includes all the primes, since after ignoring the prime number itself and 1, there are 0 possible edges (OEIS: A000961).
up to 100: 1, 2, 3, 4, 5, 7, 8, 9, 11, 13, 16, 17, 19, 23, 25, 27, 29, 31, 32, 37, 41, 43, 47, 49, 53, 59, 61, 64, 67, 71, 73, 79, 81, 83, 89, 97

IG or isolated graph:
"Squarefree semiprimes: Numbers that are the product of two distinct primes" (OEIS: A006881)
up to 100: 6, 10, 14, 15, 21, 22, 26, 33, 34, 35, 38, 39, 46, 51, 55, 57, 58, 62, 65, 69, 74, 77, 82, 85, 86, 87, 91, 93, 94, 95

LG or linear graph:
"Numbers which are the product of a prime and the square of a different prime (p^2 * q)" (OEIS: A054753)
up to 100: 12, 18, 20, 28, 44, 45, 50, 52, 63, 68, 75, 76, 92, 98, 99
Notice that according to our conjecture, all LGs have 4 divisors (e.g. the divisors of 5344962129269790721, that is a part of the sequence, are: 23, 529, 10103898164971249, 232389657794338727). The only exception of the "four divisor rule" is 16: it has four divisors (excluding 1 and the number itself), but it is a CG.

OG: other graph:
They are neither complete, nor isolated nor linear. This list differs only in the first3x places from the list of abundant numbers that contains 12,18, and 20 as well (OEIS A005101)
up to 100: 24, 30, 36, 40, 42, 48, 54, 56, 60, 66, 70, 72, 78, 80, 84, 88, 90, 96, 100

In sum: it is remarkable that there is a one-to-one correlation between certain categories of numbers and the graph structure of their divisors.

Our hypothesis is that there is at least a partial hierarchy between these categories: if a number satisfies, for example, both the CG and LG criteria, then the CG rules applies. Similarly, LG is a stronger rule than the OG (CG > LG > OG).

Obviously, it is imaginable that the connection between the categories of the numbers and graphs is only "statistical. OR:
Another possible explanation is that it is only a coincidence that the lists of LG and OG numbers are almost identical to certain OEIS lists, and a larger corpus of examples would make manifest the difference between these two category of numbers.

I prefer the hierarchy-hypothesis-not only since it is interesting, but because at least in the cases of CG and IG numbers the connection between the numbers and the types of graphs are manifest and it seems to be plausible that there is a similar logic behind the OG numbers. But further research would be needed to answer the question.

Appendix: graph numbers up to 100

number	divisors	graph-description	category
1	---	---	IG
2	---	---	IG
3	---	---	IG
4	2	---	CG
5	---	---	CG
6	2,3	---	IG
7	---	---	CG
8	2,4	2-4	CG
9	3	---	CG
10	2,5	---	IG
11	---	---	IG

12	2,3,4,6	2-4-6 3-6 (4-2-6-3)	LG
13	---	---	CG
14	2,7	---	IG
15	3,5	---	IG
16	2,4,8	2-4-8 2-8	CG
17	---	---	CG
18	2,3,6,9	2-6 3-6-9 (2-6-3-9)	LG
19	---	---	CG
20	2,4,5,10	2-4-10 5-10 (4-2-10-5)	LG
21	3,7	---	IG
22	2,11	---	IG
23	---	---	CG
24	2,3,4,6,8,12	3-6-12 4-8-12	OG
25	5	---	CG
26	2,13	---	IG
27	3,9	3-9	CG
28	2,4,7,14	2-4-14 7-14 (4-2-14-7)	LG
29	---	---	CG
30	2,3,5,6,10,15	2-6-10	OG

31	---	---	CG
32	2,4,8,16	2-4-8-16 4-8-16 8-16	CG
33	3,11	---	IG
34	2,17	---	IG
35	5,7	---	IG
36	2,3,4,6,9,12,18	2-4-6-12-18 3-6-9-12-18 4-12 6-12-18 9-18	OG
37	---	---	CG
38	2,19	---	IG
39	3,13	---	IG
40	2,4,5,8,10,20	2-4-8-10-20 4-8-20 5-10-20 10-20	OG
41	---	---	CG
42	2,3,6,7,14,21	2-6-14 3-6-21 7-14-21	OG
43	---	---	CG
44	2,4,11,22	2-4-22 11-22 (4-2-11-22)	LG
45	3,5,9,15	3-9-15 5-15 (9-3-15-5)	LG
46	2,23	---	IG
47	---	---	CG

48	2,3,4,6,8,12,16,24	2-4-6-12-16-24 3-6-12-24 6-12-24 8-24 12-24	OG
49	7	---	CG
50	2,5,10,25	2-10 5-25 (2-10-5-25)	LG
51	3,17	---	IG
52	2,4,13,26	2-4-26 13-26 (4-2-26-13)	LG
53	---	---	CG
54	2,3,6,9,18,27	2-6-18 3-6-9-18-27	OG
55	5,11	---	IG
56	2,4,7,8,14,28	2-4-8-14-28 4-8 14-28	OG
57	3,19	---	IG
58	2,29	---	IG
59	---	---	CG
60	2,3,4,5,6,10,12,15,20,30,	2-4-6-10-12-20-30 3-6-12-15-30 4-12-20 6-12-30 10-20-30 15-30	OG
61	---	---	CG
62	2,31	---	IG

63	3,7,9,21	3-9-21 7-21 (9-3-21-7)	LG
64	2,4,8,16,32	2-4-8-16-32 4-8-16-32 8-16-32 16-32	CG
65	5,13	---	IG
66	2,3,6,11,22,33	2-6-22 3-6 11-22-33	OG
67	---	---	CG
68	2,4,17,34	2-4-34 17-34 (4-2-34-17)	LG
69	3,23	---	IG
70	2,5,7,10,14,35	2-10-14 5-10-35 7-14	OG
71	---	---	IG
72	2,3,4,6,8,9,12,18,24,36	2-4-6-8-12-18-24-36 3-6-9-12-18-24-36 4-8-12-16-24-36	OG
73	---	---	CG
74	2,37	---	IG
75	3,5,15,25	3-15 5-15-25 (3-15-5-25)	LG

76	2,4,19,38	2-4-38 19-38 (4-2-38-19)	LG
77	7,11	---	IG
78	2,3,6,13,26,39	2-6-26 3-6 13-26-39	OG
79	---	---	CG
80	2,4,5,8,10,16,20,40	2-4-8-10-16-20-40 4-8-16-20-40 5-10-20-40 8-16 10-20-40 0-40	OG
81	3,9,27	3-9-27 9-27	CG
82	2,41	---	IG
83	---	---	CG
84	2,3,4,6,7,12,14,21,28,42	2-4-6-12-14-28-42 3-6-12-21-28-42 4-12-28 6-12-42 7-14-28-42 21-42	OG
85	5,17	---	IG
86	2,43	---	IG
87	3,29	---	IG
88	2,4,8,11,22,44	2-4-8-22-44 4-8-44 11-22-44 22-44	OG
89	---	---	CG

90	2,3,5,6,9,10,15,18,30,45	2-6-10-18-30 3-6-9-15-18-30-45 6-18-30 9-18-45 10-30 15-30-45	OG
91	7,13	---	IG
92	2,4,23,46	2-4-46 23-46 (4-2-46-23)	LG
93	3,31	---	IG
94	2,47	---	IG
95	5,19	---	IG
96	2,3,4,6,8,12,16,24,32,48	2-4-6-8-12-16-24-32-48 3-6-12-24-48 4-8-12-16-24-32-48 6-12-24-48 8-16-24-32-48 12-24-48 16-32-48 24-48	OG
97	---	---	CG
98	2,7,14,49	2-14 7-14-49 (2-14-7-49)	LG
99	3,9,11,33	3-9-33 11-33 (9-3-33-11)	LG
100	2,4,5,10,20,25,50	2-4-10-20-50 4-20 5-10-20-25-50 25-50	OG

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