Zoltán Galántai: New Observations on Lychrel Numbers and Palindromes

Abstract

Considering the meaning of the Lychrel number and the palindromes generated by 196 algorithm as known, the following empirical observations were made on the Lychrel-numbers:
1. Continuing the process that resulted in the first palindrome generated by the 196 algorithm starting from a certain number, probably we will find some new palindromes - but these palindrome sequences are only finite in length.
2. The palindromes generated by the 196 algorithm are clustered in bushes. In other words: certain sequences starting with different numbers end in the same palindrome.
3. There are more palindromes generated by the 196 algorithm with even than with odd ending.

Palindrome sequences

In this context, the "palindrome" means palindrome generated by the 196 algorithm (in short: algorithm). If after the appearance of the first palindrome we continue the generating process, the result is a sequence usually with a few more palindromes, e.g. examining the palindromes for 1000 times (running the palindrome searching program for 500 steps where one step is examining by the algorithm):
Starting number: 1
Palindromes: 2, 4, 8, 77, 1111, 2222, 4444, 8888, 661166, 3654563
Starting number: 2
4, 8, 77, 1111, 2222, 4444, 8888, 661166, 3654563
Starting number: 3
6, 33, 66, 363, 4884, 8836886388, 47337877873374
It is not a surprise that both 1 and 2 end in the same number since 2 is the second palindrome of the sequence beginning with 1. But it is worth mentioning that these sequences are short: for the first 100 000 numbers the longest sequence contains only 12 palindromes (with 1000 as starting number) and in the case of 10151 as starting number, 81 steps were needed to reach the last paindrome of the sequence (and it was the record). Obviously it is not known whether using 1 million or 1 trillion steps to examine the palindrome sequence we would find more paindromes, but it seems to be a plausible conjecture that these palindrome sequences are only infinitely long.

Root numbers

Using the algorithm, we can reach the same numbers as the last palindrome of the sequence that is called root number.
To give an example, up to 10 000 the next numbers as starting numers end in **4994**: 479, 578, 677, 776, 875, 974, 1093, 1183, 1273, 1363, 1453, 1543, 1633, 1723, 1813, 1903, 2092, 2182, 2272, 2362, 2452, 2542, 2632, 2722, 2812, 2902, 3091, 3181, 3271, 3361, 3451, 3541, 3631, 3721, 3811, 3901, 4090, 4180, 4270, 4360, 4450, 4540, 4630, 4720, 4810, 4900. Thus 4994 is a root number with 46 branches.

Palindromes divisible by odd numbers

There are significantly more odd endings than palindromes with even endigs and it seems to be plausible that after some early irregularities the proportion converges to a constant value somewhere between 88 and 90%.

Appendix

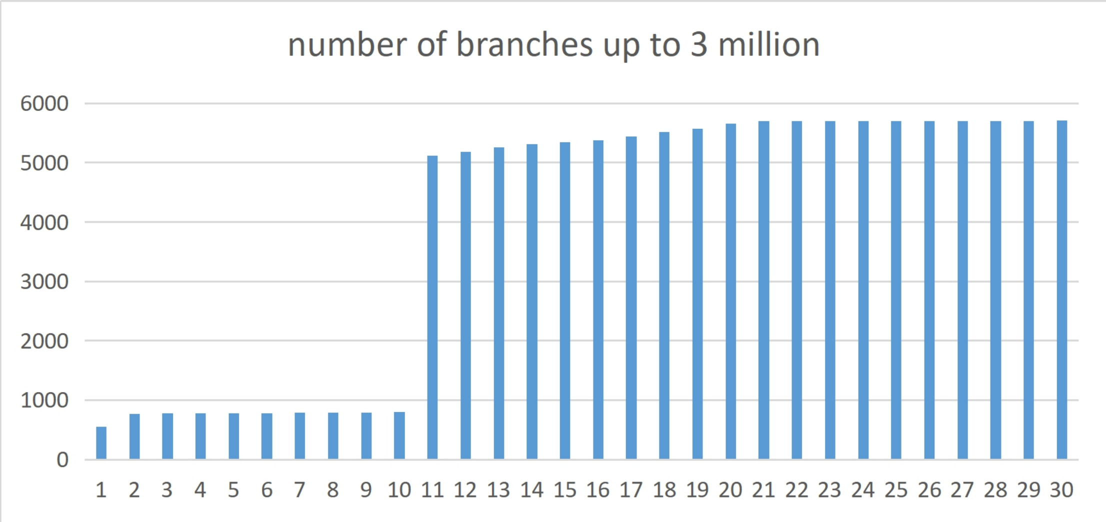
A) **The first 25 palindrome sequences for 1000 cycles:**

1	2, 4, 8, 77, 1111, 2222, 4444, 8888, 661166, 3654563
2	4, 8, 77, 1111, 2222, 4444, 8888, 661166, 3654563
3	6, 33, 66, 363, 4884, 8836886388, 47337877873374
4	8, 77, 1111, 2222, 4444, 8888, 661166, 3654563
5	11, 22, 44, 88, 44044, 88088, 44177144, 454576675454, 678736545637876

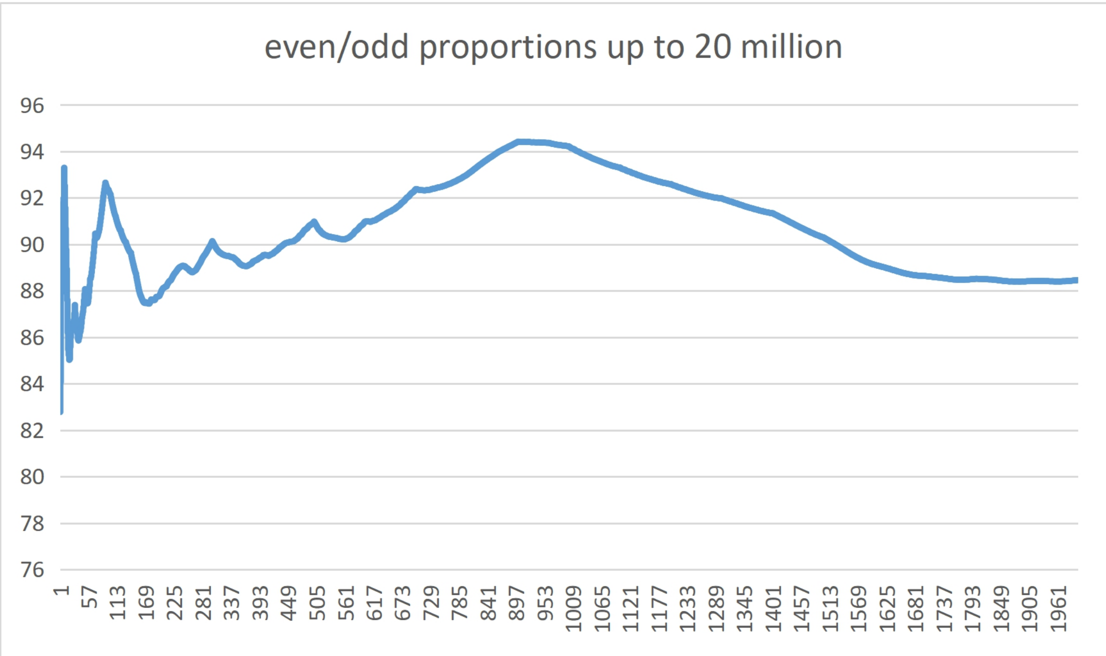
6	33, 66, 363, 4884, 8836886388, 47337877873374
7	55, 121, 242, 484, 8813200023188
8	77, 1111, 2222, 4444, 8888, 661166, 3654563
9	99, 79497, 49985258994
10	11, 22, 44, 88, 44044, 88088, 44177144, 454576675454, 678736545637876
11	22, 44, 88, 44044, 88088, 44177144, 454576675454, 678736545637876
12	33, 66, 363, 4884, 8836886388, 47337877873374
13	44, 88, 44044, 88088, 44177144, 454576675454, 678736545637876
14	55, 121, 242, 484, 8813200023188
15	66, 363, 4884, 8836886388, 47337877873374
16	77, 1111, 2222, 4444, 8888, 661166, 3654563
17	88, 44044, 88088, 44177144, 454576675454, 678736545637876
18	99, 79497, 49985258994
19	121, 242, 484, 8813200023188
20	22, 44, 88, 44044, 88088, 44177144, 454576675454, 678736545637876
21	33, 66, 363, 4884, 8836886388, 47337877873374
22	44, 88, 44044, 88088, 44177144, 454576675454, 678736545637876
23	55, 121, 242, 484, 8813200023188
24	66, 363, 4884, 8836886388, 47337877873374
25	77, 1111, 2222, 4444, 8888, 661166, 3654563

B) Number of branches up to 3 million:

Up to	**how many branches**
100000:	547
200000:	772
300000:	774
400000:	776
500000:	779
600000:	779
700000:	785
800000:	788
900000:	790
1000000:	795
1100000:	5121
1200000:	5184
1300000:	5259
1400000:	5308
1500000:	5344
1600000:	5379
1700000:	5445
1800000:	5521
1900000:	5569
2000000:	5660
2100000:	5697
2200000:	5697
2300000:	5698
2400000:	5698
2500000:	5700
2600000:	5700
2700000:	5700
2800000:	5700
2900000:	5700
3000000:	5712



C) Porportion of even / odd endings (up to 20 million)



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Sources:

Weisstein, Eric W. "Lychrel Number." From *MathWorld*--A Wolfram Web Resource.
https://mathworld.wolfram.com/LychrelNumber.html

Programs written by me.