Zoltán Galántai: Mining patterns of the digital numbers to solve equations

Constructing numbers using simple patterns of zeros and ones in binary system can make unnecessary some more or less complicated equations. Hereby I am presenting some findings as a result of a few hour search. The easiness of finding these binary patterns leads to two questions. First of all, whether it means that it is really easy to find solutions for certain types of equations or the continuing search would show that it would become more and more difficult to find new equations. My conjecture is that the scope this method is rather limited. Another question is whether there are opportunities to solve certain equations in other number systems (e.g. in ternary or hexadecimal) using this approach but I expect that the binary system is very suitable for this purpose since it contains only zeros and ones, and thus it is easy to generate not too complicated regular patterns. But in the case of the ternary system (not to mention other number systems) there are too many variability in the form of the written numbers so it is not a simple task to find patterns that generate the results of certain equations. Because of it, I focus only for the binary system now, but, obviously, a more systematic research is needed.

The method

The starting point is generating regular (symmetrical or simple and repeating) patterns using the binary notation. E.g. write down a "1", then add a "0" to it; and repeat this process for a few times:
10
1010
101010
10101010
Now we can check in OEIS database whether it is a solution of an equation (that is, presumably, based on the powers of 2 with additions, subtractions etc.) and voilà:
OEIS: A020988
a(n) = (2/3)*(4^n-1)
where the items of the list are: 2,10,42,170 etc.
It is mentioned that A020988 is about "numbers whose binary representation is 10".
In the search for the equations we can change the "10" for other patterns, e.g. 100, 101; we can change the order of the digits: instead of 10, 1010 we can play with 10, 1001, 100110 etc.

Some results – without the pursuit of completeness

rule*: start with "11", and add "11" to the last digit repeating the process to construct the next number:*
11
1111
111111
11111111
1111111111
111111111111
= 3, 15, 63, 255, 1023, 4095
OEIS: A024036
equation: a(n) = 4^n-1.

***rule**: add to the first digit (that is “1”) 1+1, 1+3; 1+5; 1+7 etc. “1”s to construct the numbers:*
1
11
1111
111111
11111111
1111111111
= 1, 3, 15, 63, 255, 1023
OEIS: A103454
equation: 0^n+4^n-1

***rule**: the first digit is “1”, the second number is constructed as 1+2 “1”s; the third number contains 1+2+3 digits… etc.*
1
111
111111
1111111111
111111111111111
111111111111111111111
= 1, 7, 63, 1023, 32767, 2097151
OEIS: A126883
equation: (2^0)*(2^1)*(2^2)*(2^3)...(2^n)-1 = 2^T_n-1 (T refers to triangular numbers)

***rule**: add to the first digit (that is “1”) 1+2; 1+2+4; 1+2+4+8 etc. “1”s to construct the numbers:*
1
111
1111111
111111111111111
1111111111111111111111111111111
111111111111111111111111111111111111111111111111111111111111111
= 1, 7, 127, 32767, 2147483647, 9223372036854775807
OEIS: A077585
equation: a(n) = 2^(2^n-1) - 1**.**

***rule**: add to the front of the number a “1” and to the end digit a “0” and repeat this process:*
10
1100
111000
11110000
1111100000
111111000000
= 2, 12, 56, 240, 992, 4032
OEIS: A020522
equation: a(n) = 4^n - 2^n.

***rule**: add a "1" to the front of the number and a "0" to the end and repeat this process:*
1
110
11100
1111000
111110000
11111100000
= 1, 6, 28, 120, 496, 2016
OEIS: A006516
equation: a(n) = 2^(n-1)*(2^n - 1), n >= 0.

***rule**: add a "1" to the front of the number and a "0" to the last "0" and repeat this process:*
11
1110
111100
11111000
11111100000
1111111000000
= 3, 14, 60, 248, 1008, 4064, 16321
OEIS: A171499
a(n) = 6*a(n-1) - 8*a(n-2) for n > 1; a(0) = 3, a(1) = 14

***rule**: add a "1" to the front of the number and a "0" to the last "0" and repeat this process:*
101
11001
1110001
111100001
11111000001
1111110000001
= 5, 25, 113, 481, 1985, 8065
OEIS: A092440
equation: a(n) = 2^(2n+1) - 2^(n+1) + 1.

***rule**: add a "1" to the last "1" and a "0" to the last "0" (notice its similarity to the previous rule):*
101
10011
1000111
100001111
10000011111
1000000111111
= 5, 19, 71, 271, 1055, 4159
OEIS: A099393
equation: 4^n + 2^n – 1

rule*: add the reverse order of the previous two digits to construct the next number:*

10
1001
100110
10011001
1001100110
100110011001
= 2, 9, 38, 153, 614, 2457
OEIS : A037489
equation: $a(n) = 4a(n-1) + a(n-2) - 4a(n-3)$

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