On the Lengths of the Look and Say Sequences

Abstract

Previous studies about the look and say sequences have focused mainly on small starting values (e.g. 1,2, or 9). Now, these sequences are examined with starting values up to 40000. The new results do not invalidate the earlier ones, but they show that

1. The growth of the lengths of the new lines of a sequence can be described by an exponential formula. Since the lengths of the sequences with different starting values produce non-totally identical growths, we have to use slightly different formulas.

2. The lengths (number of the digits) in the nth line of the sequences do not follow a simple function: their pattern is fractal-like.

1. Introduction

A look and say sentence begins with a single digit (say, 1). Then in the second line, we'll write 1,1 (since "one 1"), the third line is 2,1 ("two 1s"); the fourth is: 1,2,1,1 ("one 2, one 1"), etc. [1], [2]

NUMBER OF LINE	DIGITS OF LINE
1	1
2	11
3	21
4	1211
5	111221

In our case, the length of the look and say sequence means the length of the nth line. The lengths of the first 45 lines were examined up to 40000 as a starting value. In the case of d=1(45), that is the 45th line of the sequence with 1 as a starting value, the length is 237746. The lengths are determined by an exponential formula (see later) and, comparing them, it is rather indifferent whether the length is d=n(20), d=n(45) or d=n(100).

2. Average lengths of the individual look and say sequences

Adding together the length of the individual sequences up to d=40000(45) and dividing them by the number of examined sequences after every addition; there is a roughly logarithmic function to describe the average growth. This is not an exact formula, as there are greater or lesser irregularities in it (see table 1.). It is easy to see that as the starting value reaches infinity, the average sequence length approaches infinity as well.

Table 1. Average length for d=40000(45)

$$y = 106121\ln(x) - 14316$$
$$R^2 = 0{,}9865$$
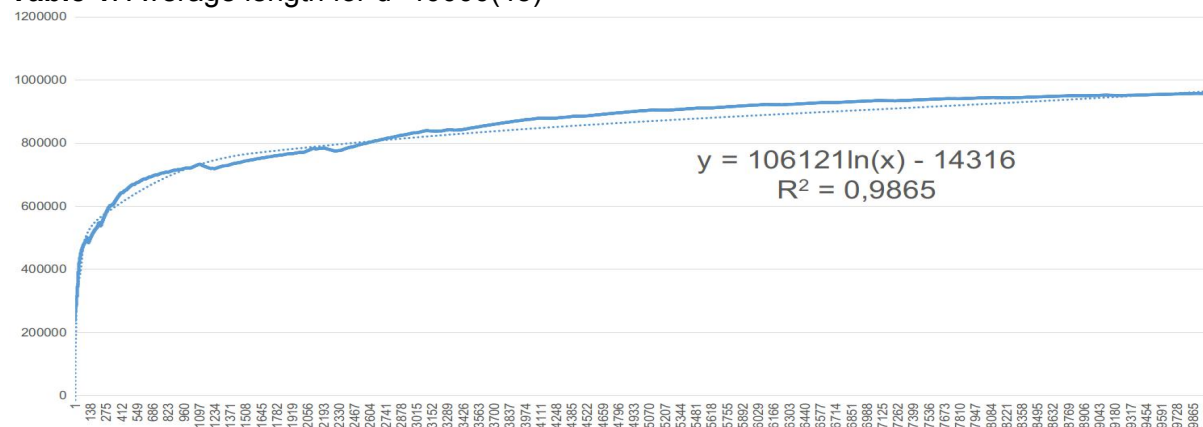
3. The growths of the lengths of the individual sequences

It follows from the above that there is no single and exact formula to describe the growth of the lengths' of the individual sequences although their formulas are similar and they seem to

follow an exponential rule (except of 22 where every line is 22). But the concrete formulas for the different sequences are slightly different.

Up to 40000, the equally shortest ones (except of 22) are 22110, 22112, 22113, 22114, 22115, 22116, 22117, 22118, and 22119. All of them have 211984 digits in the 45th line.

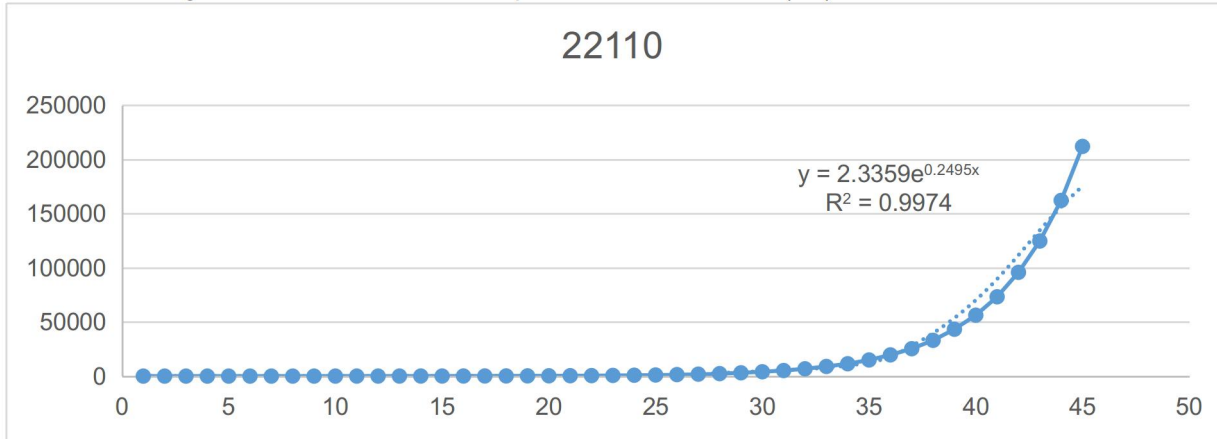**Table 2.** The growth of the shortest sequences for d=22110(45)

Table 3. The longest sequence is d=23030(45) with 1411272 digits.
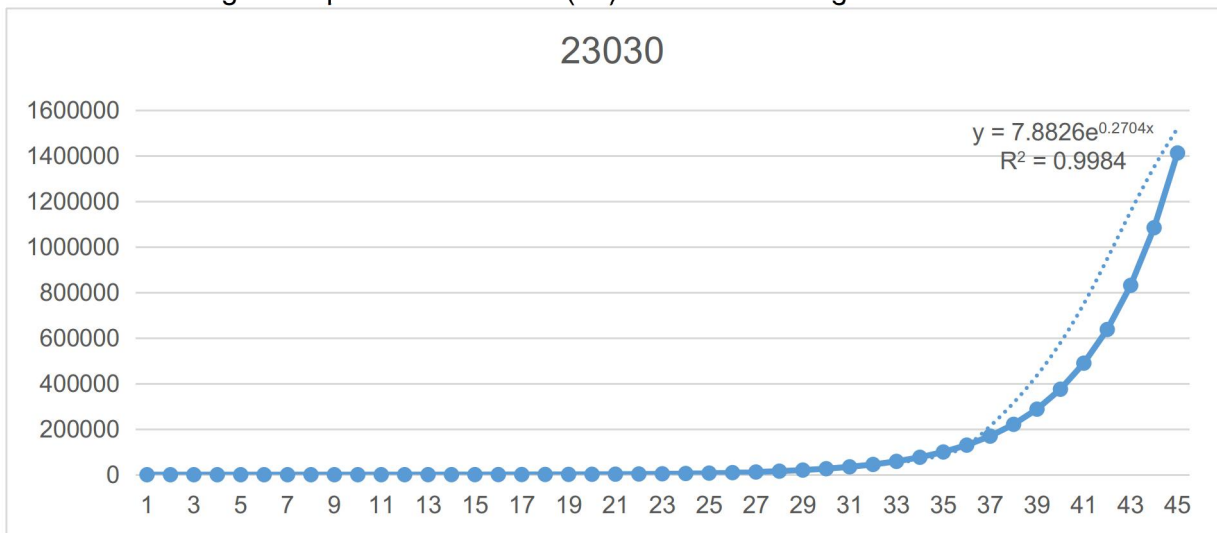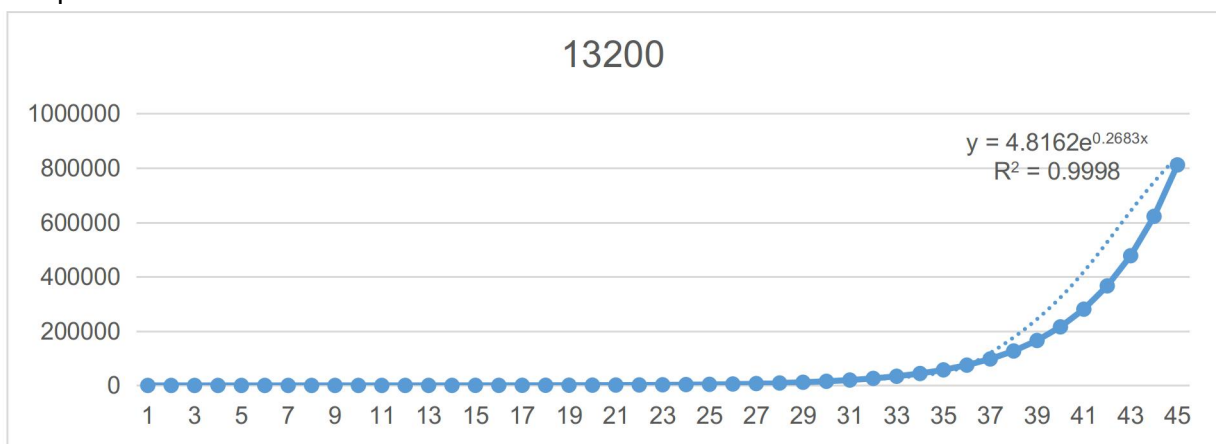

Table 4. d=13200(45): An approx. average length between the longest and shortest sequences

4. The maximum lengths of the look and say sequences

Perhaps the most surprising result is that the growth of maximum lengths of the look and say sequences follow a fractal-like, semi-regular pattern. Below this pattern is illustrated for different numbers of sequences to the 45th line.

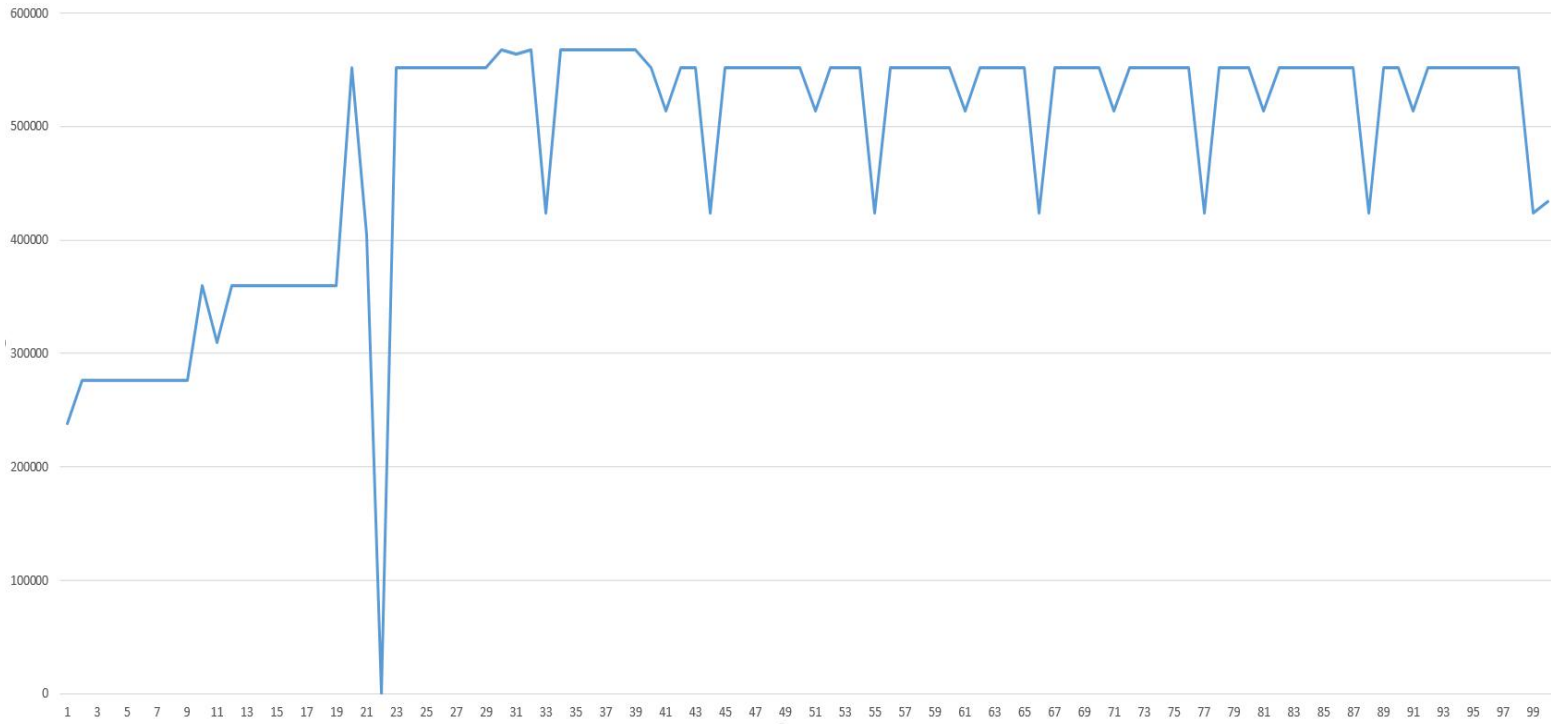**Table 5.** Maximum lengths for the first 100 look and say sequences.

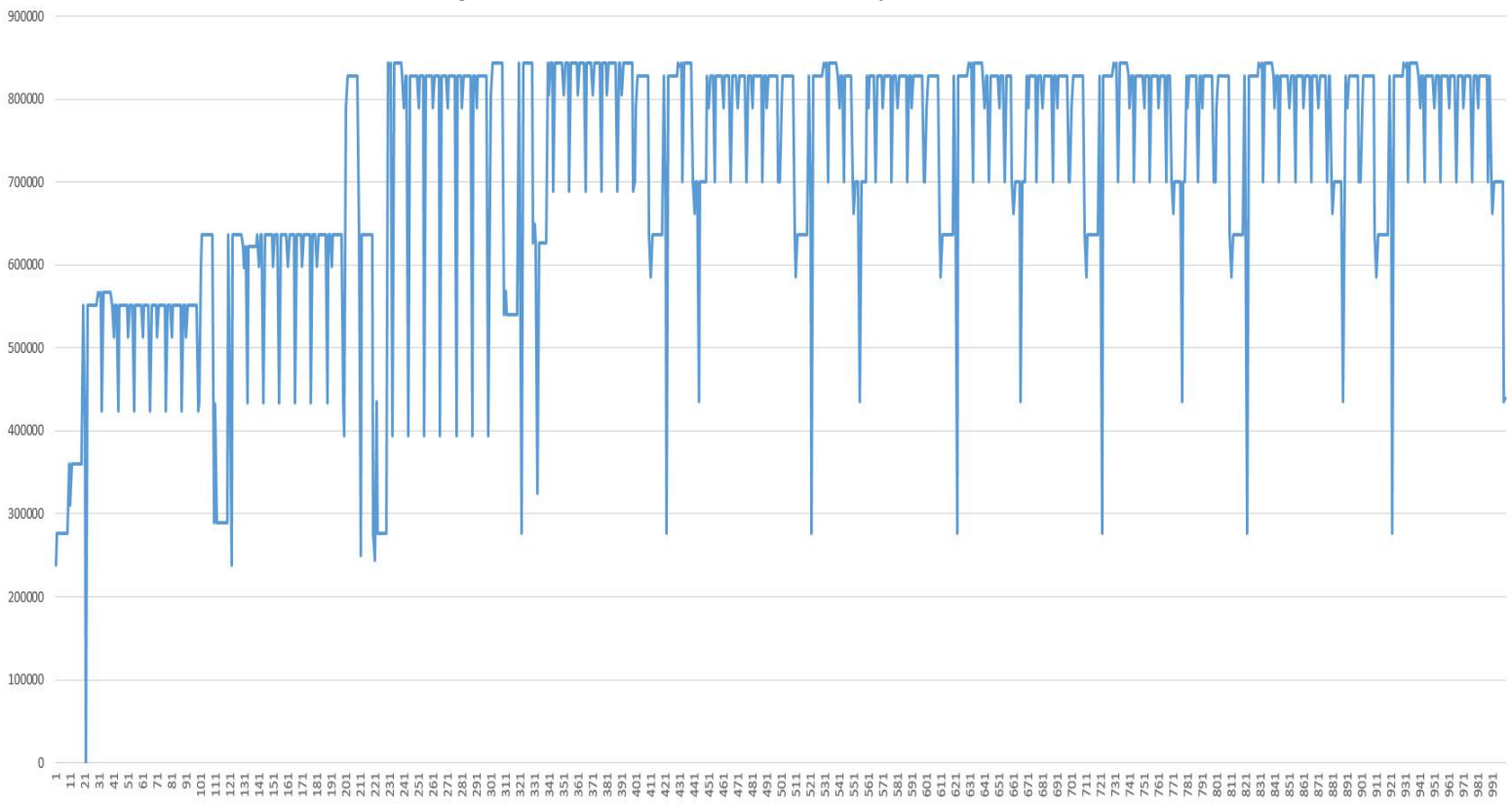**Table 6.** Maximum lengths for the first 1000 look and say sequences.

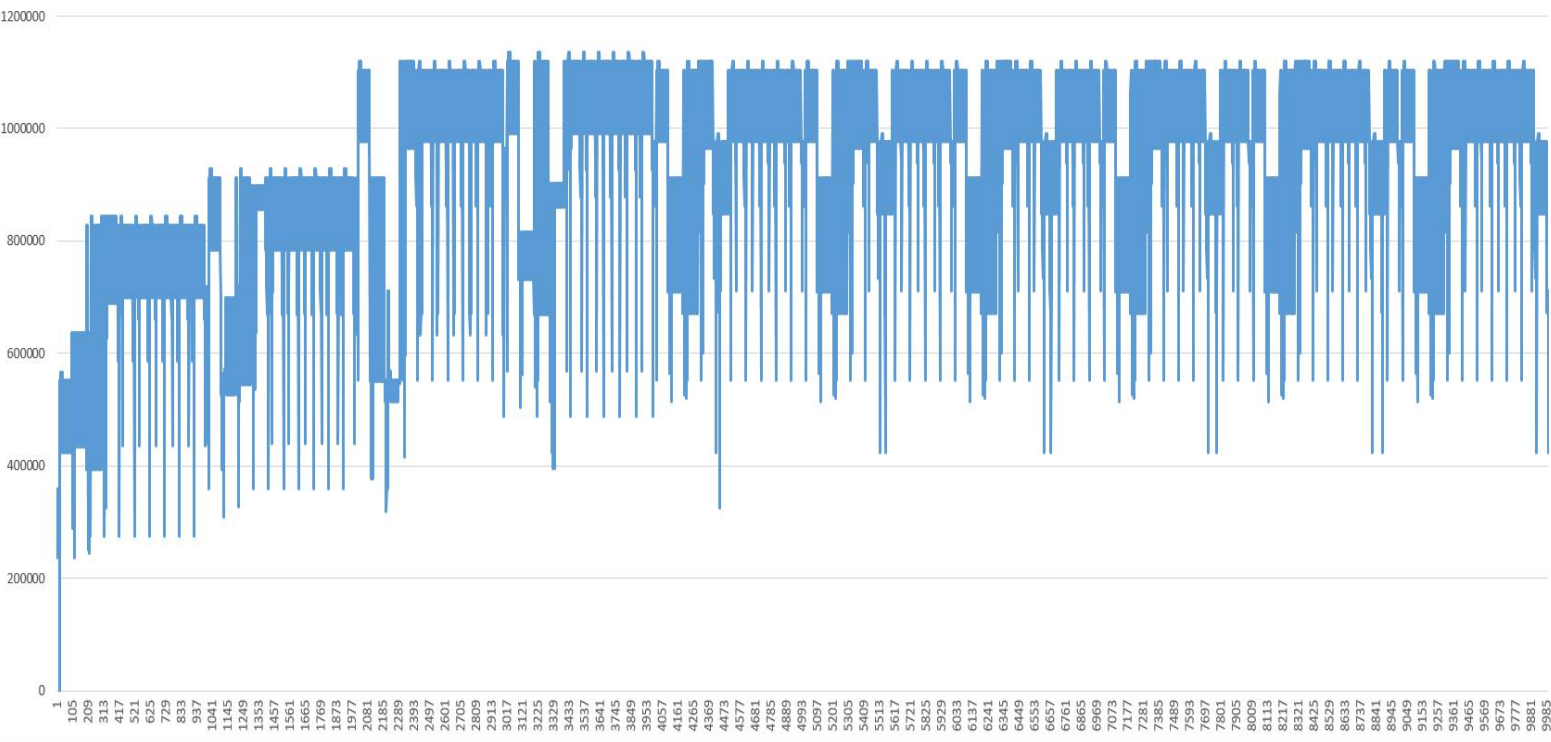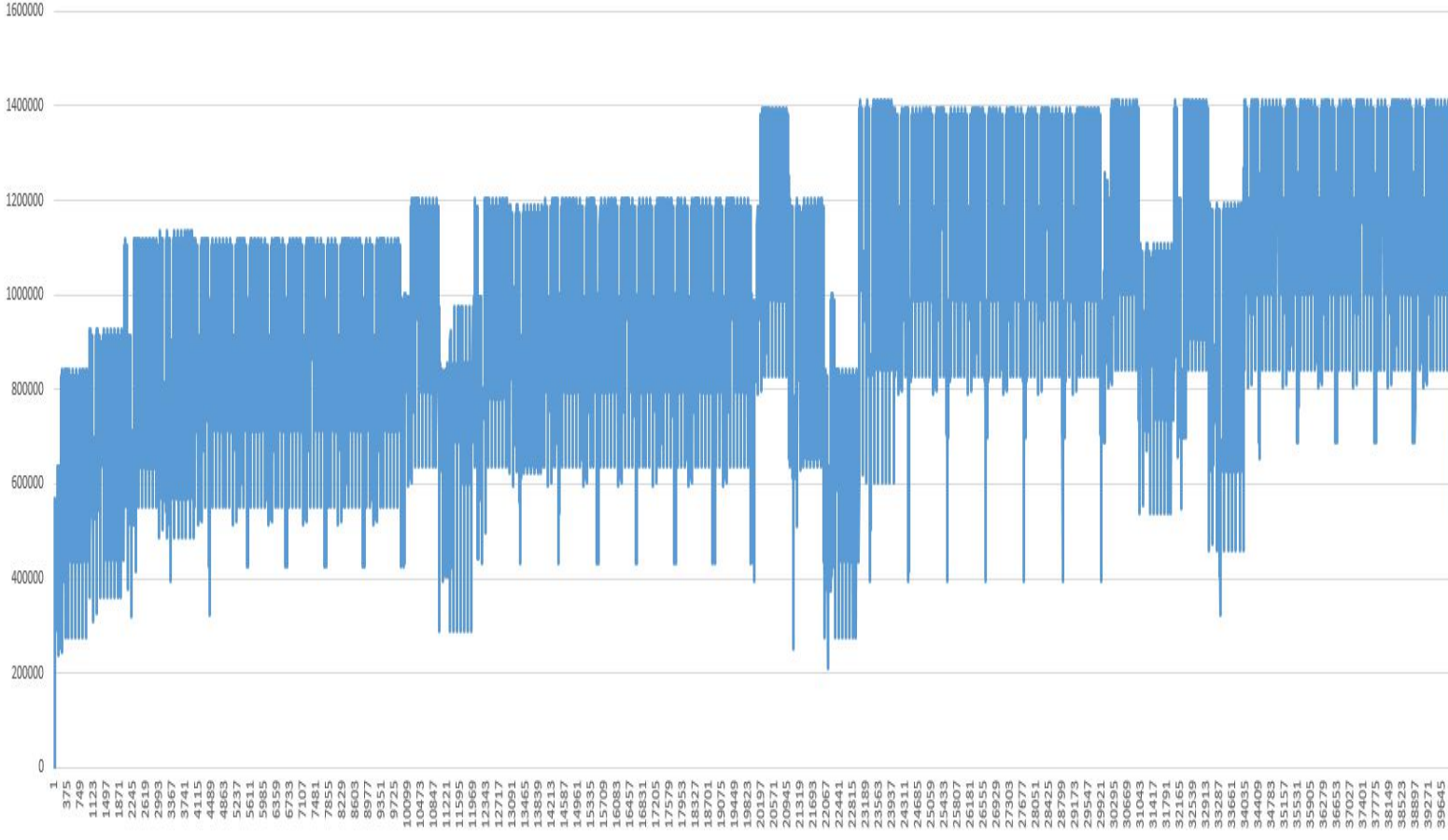**Table 7.** Maximum lengths for the first 10000 look and say sequences.

Table 8. Maximum lengths for the first 40 000 look and say sequences.

Appendix

Lengths of the first 45 line of the sequence with 13200 as starting value.
5, 8, 10, 14, 20, 26, 32, 42, 50, 68, 94, 118, 156, 214, 282, 358, 484, 634, 802, 1066, 1394, 1802, 2382, 3098, 4048, 5290, 6904, 8958, 11650, 15232, 19778, 25792, 33706, 43908, 57216, 74660, 97366, 126702, 165310, 215456, 280486, 366172, 477140, 621902, 811196

Lengths of the first 45 line of the sequence with 22110 as starting value.
5, 6, 6, 8, 10, 10, 14, 18, 20, 28, 36, 40, 52, 70, 78, 98, 138, 166, 216, 294, 376, 470, 644, 824, 1032, 1392, 1810, 2300, 3066, 4020, 5156, 6750, 8906, 11420, 14892, 19582, 25282, 33012, 43264, 56210, 73202, 95794, 124662, 162170, 211984

Lengths of the first 45 line of the sequence with 23030 as starting value.
5, 10, 20, 20, 26, 46, 60, 70, 106, 138, 166, 228, 308, 378, 494, 658, 838, 1092, 1464, 1858, 2398, 3192, 4104, 5342, 7020, 9140, 11850, 15564, 20314, 26390, 34512, 44982, 58532, 76484, 99680, 129768, 169260, 220934, 287568, 374936, 489406, 636924, 830512, 1083376, 1411272

The first 100 look and say sequence: the length of the 45th line.
237746, 275976, 275976, 275976, 275976, 275976, 275976, 275976, 275976, 360154, 310036, 360154, 360154, 360154, 360154, 360154, 360154, 360154, 360154, 551952, 403966, 2, 551952, 551952, 551952, 551952, 551952, 551952, 551952, 567648, 563348, 567648, 424096, 567648, 567648, 567648, 567648, 567648, 567648, 551952, 513722, 551952, 551952, 424096, 551952, 551952, 551952, 551952, 551952, 551952, 513722, 551952, 551952, 551952, 424096, 551952, 551952, 551952, 551952, 551952, 513722, 551952, 551952, 551952, 551952, 424096, 551952, 551952, 551952, 551952, 513722, 551952, 551952, 551952, 551952, 551952, 424096, 551952, 551952, 551952, 513722, 551952, 551952, 551952, 551952, 551952, 551952, 424096, 551952, 551952, 513722, 551952, 551952, 551952, 551952, 551952, 551952, 551952, 424096, 434180

Some more data: https://app.box.com/s/ng31jz30ciok5onuwdbhhthgu7zlcimm

Sources
[1]Conway, John H.–Guy, Richard K.: The Book of Numbers. Copernicus, 1996, p. 208–209.

[2]Weisstein, Eric W. "Look and Say Sequence." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/LookandSaySequence.html

[3] Programs were written in Python by me.

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