SHIFTED FIBONACCI-, TRIBONACCI-, N-NACCI SENTENCES; THEIR CONSTANTS AND THE RULES REGULATE THEM

Abstract

This is a summary about the shifted Fibonacci-sequences and their derivatives (Tribonacci, Tetranacci… etc. sequences) to show the existence of the constants of the shifted n-nacci sequences and to give a short description about those mathematical rules that determine the values of these constants. In sum, the shifted n-nacci sequences are governed by the same mathematical regularities as the Fibonacci sequence.

Introductory notes

Let N1, N2, N3… Nn the first, second, third… nth item of an n-nacci sequence. If we divide N2 with N1, N3 with N2, etc. then it is a shifted sequence where the degree if the shifting is 1. If N3 is divided with N1; N4 is divided with N2… the degree is 2 – and so on.
Similarly to the Golden Ratio (phi) where the constant is a result of the F2/F1; F3/F2... divisions where F is an item of the Fibonacci sequence, we can generate series constants for n-nacci sequences with different degrees. As an illustration, here are the results of the shifted Tribonacci-sequences:

degree	value
1	1.83928675521416113255
2	3.38297576790623749412
3	6.22226252312039862667
4	11.21.0497633372674333741
6	38.7165509066286292541
7	71.2108≈71.210839290136859881
8	130.977153534032922509
9	240.904543730798411645
…	

Similar lists can be generated about the Tetranacci-, Pentanacci-, n-nacci sequences, too (with different values).
„N-Step” in our case means their position in the list of the n-nacci sequences where n-Sep=1 is the Fibonacci sequence (thus the Tribonacci is the second, the Tetranacci is the third, etc.).

Values of the constants in the list of an n-nacci sequence

In the case of the Fibonacci-sequence, there is a simple rule to describe the values of the shifted constants:

degree	**constant**		**phi**
1	1.618033988749895	=	phi^1
2	2.618033988749895	=	phi^2
3	4.23606797749979	=	phi^3
4	6.854101966249685	=	phi^4
5	11.090169943749475	=	phi^5
6	17.94427190999916	=	phi^6
7	29.034441853748632	=	phi^7
8	46.97871376374779	=	phi^8
9	76.01315561749642	=	phi^9

...

It is easy to observe that the same rule valid for any n-nacci sequence, thus if c is the constant then the nth constant of the shifted sequence is c^n for any shifted n-nacci sequence.

An ad hoc example: in the case of the Octanacci shifted constants:

degree	**constant**		**c^n**
1	1.9960311797354235	=	c^1
2	3.984140470475974	=	c^2
3	7.952468603515769	=	c^3
4	15.873375288484418	=	c^4
5	31.683752003456522	=	c^5
6	63.24175688990362	=	c^6
7	126.23251861349462	=	c^7
8	251.96404304906633	=	c^8
9	502.92808609813267	=	c^9

...

Values of the constants with degree 1 of the n-nacci sequences

We can calculate the value of the 1st constants of different n-nacci sequences using a certain rule. In this case, also the Fibonacci sequence serve s as a model where the value of phi is a real root of the equation of 0=x^2 - x -1.

n-Step	**Value of**	**1st degree constant:**
1	Fibonacci =	1.618033988749895
2	Tribonacci =	1.83928675521416113255
3	Tetranacci =	1.92756197548292530426
4	Pentanacci =	1.96594823664548533718
5	Hexanacci =	1.9835828434243263
6	Heptanacci =	1.991964196605035
7	Octanacci =	1.9960311797354235

…

This values can be generated using by the following rule:

name	**n-Step**									
Fibonacci	**1**							x^2	-x	-1
Tribonacci	**2**						x^3	$-x^2$	-x	-1
Tetranacci	**3**					x^4	$-x^3$	$-x^2$	-x	-1
Pentanacci	**4**				x^5	$-x^4$	$-x^3$	$-x^2$	-x	-1
Hexanacci	**5**			x^6	$-x^5$	$-x^4$	$-x^3$	$-x^2$	-x	-1
Heptanacci	**6**		x^7	$-x^6$	$-x^5$	$-x^4$	$-x^3$	$-x^2$	-x	-1
Octanacci	**7**	x^8	$-x^7$	x^6	$-x^5$	$-x^4$	$-x^3$	$-x^2$	-x	-1

Since the first n-nacci sequence (n-Step = 1) is the Fibonacci sequence and the second (n-Step = 2) one is the Tribonacci one, etc. the rule is clear: the largest exponent of x is n-Step+1 and the constant is a real root of $0 = x^{n\text{-}Step+1} - x^{n\text{-}Step} \ldots - x^1 - x^0$.

Summary

1. The value of the nth constant is: c^n
for any shifted n-nacci sequence (where c is the constant of the first-degree sequence, e.g. in the case of the Fibonacci sequence this is phi; and n is the degree of the sequence.
2. $0 = x^{n\text{-}Step+1} - x^{n\text{-}Step} \ldots - x^1 - x^0$
is the equation that determines the values of the constants (where n-Step is 1 for the Fibonacci; 2 for the Tribonacci sentence etc.

Appendix: constants of some shifted sequences

Notice that the first-degree constants converge to 2:

Tribonacci: 1.83928675521416113255
Tetranacci: 1.9275619754829254
Pentatnacci: 1.96594823664548533718
Hexanacci: 1.9835828434243263
Heptanacci: 1.991964196605035
Octanacci: 1.9960311797354235

Thus the second-degree constants converge to 4 (2^2) and so on.

Tribonacci:

1 1.83928675521416113255
2 3.38297576790623749412
3 6.22226252312039862667
4 11.44452504624079725334
5 21.0497633372674333741
6 38.7165509066286292541
7 71.210839290136859881
8 130.977153534032922509
9 240.904543730798411645
10 443.09253655496819403

Tetranacci:

1 1.9275619754829254
2 3.7154951693276375
3 7.161847208486447
4 13.80490435329701
5 26.60980870659402
6 51.292055437705116
7 98.8686157060826
8 190.57538420367874
9 367.3458640540605
10 708.0819194015269

Penatnacci:

1	1.96594823664548533718
2	3.86495246916949321644
3	7.598296491482379721
4	14.93785758893936241175
5	29.3670547862367206870
6	57.7341095724734413741
7	113.5022709083014
8	223.13958934743331
9	438.6808822033843
10	862.4239068178291

Hexanacci:

1	1.9835828434243263
2	3.9346008967273347
3	7.80460683447031
4	15.481084216527549
5	30.708013049511177
6	60.9118878406607
7	120.8237756813214
8	239.66396851921846
9	475.3933361417096
10	942.9820654489489

Heptanacci:

1	1.991964196605035
2	3.9679213605563426
3	7.903957285172573
4	15.744399923559298
5	31.36228094476117
6	62.47254076583258
7	124.44306447648701
8	247.88612895297402
9	493.78029370934297
10	983.5926660581296

Octanacci:

1	1.9960311797354235
2	3.984140470475974
3	7.952468603515769
4	15.873375288484418
5	31.683752003456522
6	63.24175688990362
7	126.23251861349462
8	251.96404304906633
9	502.92808609813267
10	1003.86014101653

Sources

A000045 Fibonacci numbers

A000073 Tribonacci numbers

A000078 Tetranacci numbers

A001591 Pentanacci numbers

A001592 Hexanacci numbers

A079262 Octanacci numbers

Noe, Tony; Piezas, Tito III; and Weisstein, Eric W. "Fibonacci n-Step Number." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/Fibonaccin-StepNumber.html

Besides the programs written by me, Wolfram Alpha was used for some calculations.

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